



**CONCOURS D'ADMISSION
SERIE GCE**

MATHEMATICS TEST
TIME : TWO HOURS

Exercise 1 : 5 Marks

- 1) Calculate each of the following integrals : $\int_0^{\ln 2} \frac{e^x}{1+e^x} dx$ and $\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{x}{1+x^2} dx$. **2mrks**
- 2) Calculate the following limit $\lim_{n \rightarrow +\infty} \sum_{k=1}^n \frac{1}{k(k+1)}$. **1mrk**
- 3) Solve the equation $z^2 + z + 4 - 2i = 0$. in the set of complex numbers **1mrk**
- 4) Give the exponential form of the complex number $z = \frac{1+e^{i\frac{\pi}{4}}}{1-e^{i\frac{\pi}{4}}}$. **1mrk**

Exercise 2 : 4 Marks

An umbrella vendor opens his shop 250 days a year and on these days, there are 75 rainy days, of rainy weather, 50 days of moody weather and 125 sunny days. An analysis of his sales gave the following results:

- On a sunny day, there is an 80% chance of not selling an umbrella and a 20% chance of selling just one umbrella.
- On a moody day, there is a 30% chance of not selling an umbrella, 50% chance of selling just one umbrella and 20% chance of selling exactly two umbrellas.
- On a rainy day, there is a 20% chance of selling just one umbrella, 50% chance selling exactly two umbrellas and 30% chance of selling exactly three umbrellas.

Let X be the random variable « the number of umbrellas sold on a particular day ».

- 1) What is the probability that the weather will be gloomy one of the sales days? **0.5mrk**
- 2) Giving that the weather is moody, what is the probability of selling at least one umbrella? **0.5mrk**
- 3) Giving that it is rainy, what is the probability of selling at most an umbrella? **0.5mrk**
- 4) State the probability distribution of X. **1mrk**
- 5) Calculate the expectation of X. **0.5mrk**
- 6) Given that the vendor has sold just one umbrella, what is the probability that it is a rainy day? **1mrk**

Problem : 11 Marks

Given a real-valued function f defined by $f(x) = \ln(2x + \sqrt{1 + 4x^2})$ and (C_f) its curve in the xy -plane

- 1) Solve the inequality $2x + \sqrt{1 + 4x^2} > 0$. in the set of all real numbers **1mrk**
- 2) Deduce that the function f is defined on \mathbb{R} and calculate the limit of f at the bounds of \mathbb{R} . **1,5mrk**
- 3) Show that function f is odd. **0,5mrk**
- 4) Study the differentiability of f on \mathbb{R} , determine its derivative f' and give the sense of the variation of f .
1mrk
- 5) Draw the variation table of f . **0,75mrk**
- 6) Write the equation of the tangent (T) to curve (C_f) at the point where the abscissa is zero. **0,75mrk**
- 7) Study the infinite pieces of (C_f) . **0,75mrk**
- 8) Sketch (C_f) and (T) . **1,25mrk**
- 9) Show that the function f assumes a mutual bijection g defined on an interval J to be determined.
0,75mrk
- 10) Draw in broken lines the curve of the function g on the xy -plane above **0,75mrk**
- 11) Show that for every x in J , $g(x) = \frac{e^x - e^{-x}}{4}$. **0,5mrk**
- 12) Given that $h(x) = \frac{g(x)}{g'(x)}$.
 - a. Show that function h is a bijection on the set on real numbers around an interval K that will be determined. **1mrk**
 - b. Give the set on which h^{-1} is differentiable, hence express $(h^{-1})'(x)$ with respect to x **0,5mrk**