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# COMPETITIVE ENTRANCE EXAMINATION
## SERIE : GCE/AL

**SUBJECT of Mathematics**
**TIME : TWO HOURS**

**Exercise 1 :( 4 marks)**

Three tickets labeled 1, 2 and 3 are placed in three opaque identical envelops. The three envelops containing the tickets are mixed together with five other empty envelops of the same kind. The eight envelops are selected at random one after the other without replacement.

1. What is the probability that: the first envelop selected contains ticket n° 1, the second envelop selected contains ticket n° 2, and the third envelop selected contains ticket n° 3? **(1 mark)**
2. What is the probability that: the first envelop selected is empty? **(1 mark)**
3. Let X be the order of selection of the first empty envelop. Determine the probability law of the random variable X. **(2 marks)**

**Exercise 2: (5 marks)**

A. Calculate each of the following integrals: $\int_0^{\ln 2} \frac{e^t}{1+e^t}dt$ and $\int_{\frac{\pi^2}{4}}^{\frac{\pi^2}{9}} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ **(2marks)**

B. Each of the following three questions has four answers of which only one is correct. Choose the letter corresponding to the right answer for each question.

1. The expression $e^{(2-i\alpha)}$ is equal to:

A. $e^{\alpha}+ie^{\alpha}$ B. $e^{\alpha}\cos\alpha - i\sin\alpha$ C. $e^{2\alpha}\left(\cos 2\alpha - i\sin 2\alpha\right)$ D. $e^{2\alpha}\left(\cos\alpha - i\sin\alpha\right)$ **(1 mark)**

2. The algebraic form of $\frac{4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is:

A.) $\sqrt{2}+i\sqrt{6}$ B.) $\sqrt{6}+i\sqrt{2}$ C.) $\sqrt{2}+\sqrt{6}+i\left(\sqrt{2}-\sqrt{6}\right)$ D.) $\sqrt{2}-\sqrt{6}+i\left(\sqrt{2}-\sqrt{6}\right)$ **(1 mark)**

3. A player rolls a fair die with faces labeled 1 to 6. He scores 10 points if the die shows a 6 up. He scores 1 point if the die shows 1 or 4 up. He scores no point in all other cases. Let Y be the random variable, "the score of the player at each roll of the die". The expectation of Y is equal to:

A.) 2 B.) 7 C.) –2 D.) 13 **(1 mark)**

**Problem: (11 marks)**

Consider the functions $f$ and $g$ defined on $]0;+\infty[$ by $f(x)=2x+3+\frac{\ln x}{x}$ and $g(x)=2x^2+1-\ln x$.

Let $(C)$ represents the curve of the function $f$ in the xy-coordinate plane.

1. (a) Calculate the limits of the function $g$ in $0^+$ and in $+\infty$. **(1 mark)**

   (b) Find the derivative $g'$ of the function $g$ and hence draw the table of variations of $g$. **(1.5 marks)**

   (c) Deduce the sign of $g(x)$ in accordance with the values of $x$. **(0.5 marks)**

2. (a) Calculate the limits of $f$ in $0^+$ and in $+\infty$. **(0.5 marks)**

   (b) Deduce that the function $f$ assumes an asymptote parallel to a coordinate axis to be determined. **(0.5mark)**

   (c) Show that the straight line $(D)$ denoted by the equation $y = 2x - 3$ is an asymptote to $(C)$ and hence state the relative position of $(C)$ and $(D.)$ **(1 mark).**

3. (a) Show that for all $x$ in $]0, +\infty[$ $f(x) = \dfrac{g(x)}{x}$ **(1 mark)**

   (b) Deduce the sense of the variation of $f$ and hence draw the table variations of $f$. **(1 mark)**

4. Write down the equation of the tangent to $(C)$ at the point where the abscissa is 1. **(1 mark)**

5. Sketch $(C)$ and $(D)$. **(1.5 marks)**

6. Calculate the area of the region bounded by $(C)$, the straight line $(D)$ and the ordinates $x = 1$ and $x = e$. **(1 mark)**